

# Scalar field with the source in the form of the stress-energy tensor trace as a dark energy model

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We consider a scalar-tensor theory of gravitation with the scalar source being the trace of the stress-energy tensor of the scalar field itself and matter. We obtain an example of a numerical solution of the cosmological equations which shows that under some special choice of the scalar parameters, there exists a slow-roll regime in which the modern values of the Hubble and deceleration parameters may be obtained.

## I. INTRODUCTION

As well known, that the accelerated expansion of the Universe is successfully described by the  $\Lambda$ CDM model. However, the nature of the  $\Lambda$ -term is unclear and the physical interpretation of this constant led us to the famous problem of 120 orders [1, 2]. For this reason, in the literature are discussed various alternative models of the dark energy. Scalar fields are often considered as candidates for the dark energy (see, for example, [3–6] or brief but enough exhausting review in [7]). Nevertheless, there is no unambiguous criterion for the choice of the field Lagrangian in the scalar field theories. Moreover, as it was shown in [8], any scalar field in the slow-roll regime can simulate the cosmological constant and hence lead to an appropriate cosmological scenario.

As known, the Einstein equations of the gravitational field can be derived if one considers a tensor field in Minkowski space-time with the source being the total stress-energy tensor of both field and matter [9, 10]. In the present work, we consider a scalar field with a similar property: assuming that the scalar field source is the trace of the stress-energy tensor of both matter and the field itself and that this scalar field minimally interacts with gravitational field. In the frameworks of this approach derive receiving the scalar field Lagrangian which contains three parameters: new constant of the scalar interaction, scalar field mass and a parameter which relates to the minimum of scalar potential [11]. Note that the task to get such free scalar Lagrangian was considered many years ago by P. Freund and Y. Nambu in [12]. Our Lagrangian generalizes that one from [12] and besides we consider the interaction of the scalar field with the matter.

## II. SCALAR FIELD EQUATION

In this article, we use the gravitational system of units  $G = c = 1$ . Let us consider a scalar field  $\varphi$  with the

source being the total trace of the stress-energy tensor of the scalar field and matter fields. This condition implies that the scalar field equation has the following form

$$(\square - m^2)\varphi = q(T^\varphi + T^M), \quad (1)$$

the d'Alembertian is  $\square = -\nabla_\mu \nabla^\mu g^{\mu\nu}$ , the constants  $m$  and  $q$  mean the mass and the interaction constant of the scalar field<sup>1</sup>.

Equation (1) allows to determine the Lagrangian of the scalar field and matter [11]

$$L_\varphi + L_M = \frac{1}{2} \left( \frac{\partial_\mu \varphi \partial^\mu \varphi}{1 + 2q\varphi} - m^2 \varphi^2 + C(1 + 2q\varphi)^2 \right) \sqrt{-g} + L_M((1 + 2q\varphi) g_{\mu\nu}, Q_M), \quad (2)$$

where  $Q_M$  stands for matter fields. Note that in [12] the term with constant  $C$  was absent. The total Lagrangian of the model with minimal interaction between scalar and gravitational field takes the form

$$L = L_g + L_\varphi + L_M, \quad (3)$$

where

$$L_g = R\sqrt{-g} \quad (4)$$

is the Lagrangian of gravitational field.

Lagrangian (2) has non-linear kinetic energy term and belong to so-called k-essence model [13].

The scalar field equation and his stress-energy tensor have the form

$$\varphi_{;\mu}^{\cdot\mu} + m^2 \varphi = q \left( \frac{\varphi_{;\mu} \varphi^{\cdot\mu}}{1 + 2q\varphi} - 2m^2 \varphi^2 + 2C(1 + 2q\varphi)^2 - T^M \right), \quad (5)$$

$$T_\varphi^{\mu\nu} = \frac{\partial^\mu \varphi \partial^\nu \varphi}{\Phi} - \frac{1}{2} g^{\mu\nu} \left( \frac{\partial_\alpha \varphi \partial^\alpha \varphi}{\Phi} - m^2 \varphi^2 + C\Phi^2 \right). \quad (6)$$

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<sup>1</sup> The metric signature  $(+, -, -, -)$  is used

So, the interaction between the matter and the scalar field is realized with the help of the effective metric

$$f_{\mu\nu} = (1 + 2q\varphi)g_{\mu\nu} = \Phi g_{\mu\nu}. \quad (7)$$

The scalar field, interacting with the matter, was investigated in several papers (see, for example, [14, 15]), in particular within the framework of chameleon model [16, 17].

Let us consider the scalar field in the state with the minimal energy. The minimum of the energy is achieved at the point where the potential

$$V(\varphi) = \frac{1}{2}m^2\varphi^2 - \frac{C}{2}(1 + 2q\varphi)^2 \quad (8)$$

has a minimum. Here  $\varphi$  is assumed to vary from  $-1/2q$  to  $+\infty$ , which ensures the positivity of the denominator in Lagrangian (2). Under the condition  $C < m^2/4q^2$ , potential has the minimum at the point

$$\varphi_0 = \frac{2qC}{m^2 - 4q^2C}, \quad (9)$$

$$V(\varphi_0) = -\frac{m^2C}{2(m^2 - 4q^2C)}. \quad (10)$$

For  $C < 0$ , the minimum value of the potential is positive:  $V(\varphi_0) > 0$ , therefore it can be identified with the cosmological constant in the Einstein equations.

For  $0 < C < m^2/4q^2$ , the minimum value of the potential is negative:  $V(\varphi_0) < 0$ . In this case,  $|V(\varphi_0)|$  can be interpreted as the squared graviton mass  $\mu^2$ , since the Einstein equations in the linear approximation can be written as [11]

$$(\square + V(\varphi_0))\psi_{\mu\nu} = V(\varphi_0)\gamma_{\mu\nu}. \quad (11)$$

Here, in contrast to the free massive Fierz-Pauli equation in the Minkowski space, the term  $V(\varphi_0)\gamma_{\mu\nu}$  is the stress-energy tensor of the scalar field in the ground state.

Assuming that  $|C|$  is small as compared with  $m^2/q^2$  we get the estimation  $\mu^2 \approx |C|/2$ .

### III. COSMOLOGICAL SOLUTION

Let us consider the application of the scalar-tensor theory of gravity to cosmology. As a first approximation we consider the field equations without interaction of the scalar field with matter. Thus the Lagrangian of the matter takes the form

$$L_M(f_{\mu\nu}, Q_M) \rightarrow L_M(g_{\mu\nu}, Q_M). \quad (12)$$

The scalar factor vanishes from stress-energy tensor of the matter in the equations of gravitational, field and

stress-energy tensor trace vanishes from scalar field equation. Thus system of equations takes the form

$$G^{\mu\nu} = 8\pi\left(\frac{\partial^\mu\varphi\partial^\nu\varphi}{\Phi} - \frac{1}{2}\frac{\partial_\alpha\varphi\partial^\alpha\varphi}{\Phi}g^{\mu\nu} + \frac{1}{2}m^2\varphi^2g^{\mu\nu} - \frac{1}{2}C\Phi^2g^{\mu\nu} + (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}\right), \quad (13)$$

$$(\square - m^2)\varphi = q\left(-\frac{\partial_\mu\varphi\partial^\mu\varphi}{\Phi} + 2m^2\varphi^2 - 2C\Phi^2\right), \quad (14)$$

Spatially flat homogeneous and isotropic universe is described by Friedman-Lemaitre-Robertson-Walker the metric

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2). \quad (15)$$

Cosmological equations, which include Friedman equation, massless scalar field equation and covariant law of conservation of stress-energy tensor of matter, read

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\left(\frac{1}{2}\frac{\dot{\varphi}^2}{\Phi} + \frac{1}{2}m^2\varphi^2 - \frac{1}{2}C\Phi^2 + \epsilon\right), \quad (16)$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{q\dot{\varphi}^2}{\Phi} + m^2\varphi\Phi - 2Cq\Phi^2 = 0, \quad (17)$$

$$\dot{\epsilon} + 3H(\epsilon + p) = 0. \quad (18)$$

Here  $H \equiv \dot{a}/a$  is the Hubble parameter.

Let us consider the system of equations with the massless scalar field (16–18). Comparing the stress-energy tensor of the scalar field with stress-energy tensor of the ideal fluid, we can write the following relations

$$\epsilon^\varphi = \frac{1}{2}\left(\frac{\dot{\varphi}^2}{\Phi} - C\Phi^2\right), \quad (19)$$

$$p^\varphi = \frac{1}{2}\left(\frac{\dot{\varphi}^2}{\Phi} + C\Phi^2\right). \quad (20)$$

If we identify the density of the dark energy at present stage with the energy density of the scalar field and specify parameter  $\omega$  in the equation of state we can get the initial values  $\varphi_0$  and  $\dot{\varphi}_0$  for the equation of the scalar field

$$\epsilon_0^\varphi(1 + \omega) \equiv a = \frac{\dot{\varphi}_0^2}{\Phi_0}, \quad (21)$$

$$\epsilon_0^\varphi(1 - \omega) \equiv b = -C\Phi_0^2, \quad (22)$$

where  $\Phi_0 = 1 + 2q\varphi_0$ . Further, we will omit the index 0. One can consider two cases, which correspond to different parameters  $\omega - \omega > -1$  and  $\omega < -1$ . Then from the system of equations (21), (22) we obtain two solutions

$$\omega > -1, \quad a > 0, \quad \varphi_q \geq -\frac{1}{2q} \quad (23)$$

and

$$\omega < -1, \quad a < 0, \quad \varphi_{ph} \leq -\frac{1}{2q} \quad (24)$$

This case corresponds to so-called phantom dark energy [19, 20].

From equation (22) we get

$$\varphi_{1,2} = -\frac{1}{2q} \pm \frac{1}{2q} \sqrt{\frac{b}{C'}}, \quad (25)$$

which implies that  $C < 0$ , and  $C' \equiv -C > 0$ . For  $\dot{\varphi}$  we have

$$\dot{\varphi} = \pm \left( \pm a \sqrt{\frac{b}{C'}} \right)^{\frac{1}{2}}. \quad (26)$$

Taking into account the inequalities (23) and (24), we obtain the following initial values  $\varphi$  and  $\dot{\varphi}$  for the  $\omega > -1$ :

$$\varphi_q = -\frac{1}{2q} + \frac{1}{2q} \sqrt{\frac{\epsilon(1-\omega)}{C'}}, \quad (27)$$

$$\dot{\varphi}_q = \left( \epsilon(1+\omega) \sqrt{\frac{\epsilon(1-\omega)}{C'}} \right)^{\frac{1}{2}}, \quad (28)$$

and for the  $\omega < -1$ :

$$\varphi_{ph} = -\frac{1}{2q} - \frac{1}{2q} \sqrt{\frac{\epsilon(1-\omega)}{C'}}, \quad (29)$$

$$\dot{\varphi}_{ph} = \left( -\epsilon(1+\omega) \sqrt{\frac{\epsilon(1-\omega)}{C'}} \right)^{\frac{1}{2}}. \quad (30)$$

#### IV. COSMOLOGICAL SCENARIO INVESTIGATION

At present stage, a scalar field must be in the slow roll mode when the following relation is satisfied:

$$3H\dot{\varphi} = 2Cq\Phi = V'\Phi, \quad (31)$$

where  $V' \equiv \frac{\partial V}{\partial \varphi}$ . Expression (31) was obtained under the assumption that  $q\dot{\varphi}^2/(1+2q\varphi)$  is small. This term can be neglected in comparison with  $3H\dot{\varphi}$  because of smallness of  $q$  and  $\dot{\varphi}^2$ . This slow roll mode can be used for

establishing relation between constants  $C$  and  $q$ . One can substitute the initial values of  $\varphi$  and  $\dot{\varphi}$  in (31). Then, using the dimensionless quantities  $\overline{C}$  and  $\overline{\epsilon}_\varphi$ , we obtain an expression binding the constants  $\overline{C}$  and  $q$

$$\overline{C}' = \frac{C'}{H^2} = \frac{81(1+\omega)^2}{16q^4(1-\omega)^3\overline{\epsilon}_\varphi}, \quad (32)$$

where  $\overline{\epsilon}_\varphi \equiv \epsilon_\varphi/H^2$  dimensionless energy density of the scalar field. The relation (31) allows us to establish the sign of the initial value  $\dot{\varphi}$ , namely  $\dot{\varphi} < 0$ . Thus with an appropriate choice of the constant  $q$ , the slow-rolling is provided automatically both in two models of the dark energy. The rolling speed  $\dot{\varphi}$  depends only on  $q$ , as follows from (26) and (32).

For the numerical solving of system of equations (16–18) we have to pass to dimensionless quantities in this system. Let us define the dimensionless time as  $T = tH_0$ . Hence  $T = 1$  means a period of time equal to one age of the Universe. Then the system of cosmological equations takes the form

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi}{3} \left( \frac{(\varphi')^2}{2\Phi} - \overline{C}\Phi^2 + \overline{\epsilon} \right), \quad (33)$$

$$\varphi'' + 3\overline{H}\varphi' - q\frac{(\varphi')^2}{\Phi} + 2\overline{C}q\Phi^2 = 0, \quad (34)$$

$$\epsilon' + 3\overline{H}(\epsilon + p) = 0. \quad (35)$$

We have one arbitrary parameter  $q$ . The numerical solution of the cosmological equations we will obtain when the value of the parameter equals  $q = 1$ . In SI units this value is several orders of magnitude less than the gravitational constant which ensures the absence of symptoms of a scalar field in the solar system. The numerical solution of these equations is shown in figures.

The figure 1 shows decreasing the energy density of scalar field and matter with time. Radiation energy density can be neglected because its contribution to the total energy density of the universe is very small in the considered period of time. Moment of nearly equal contribution of energy densities of scalar field and matter corresponds to dimensionless time  $T = 0.672$  and red-shift  $z = 0.431$ . The same values for  $\Lambda$ CDM model are  $T = 0.68$  and  $z = 0.417$ .

From the figures 2 and 3 we see that the value of the Hubble parameter at the present time which follows from our model coincides with the observed value of Hubble constant and value which follows from  $\Lambda$ CDM model. In dimensionless units these values corresponds to  $(\overline{H}(1) = H(1)/H_0 = 1)$ . However previous and further evolution of Hubble parameter in different models is different. From  $H(z)$  dependence one can see that our model gives much bigger Hubble parameter in the

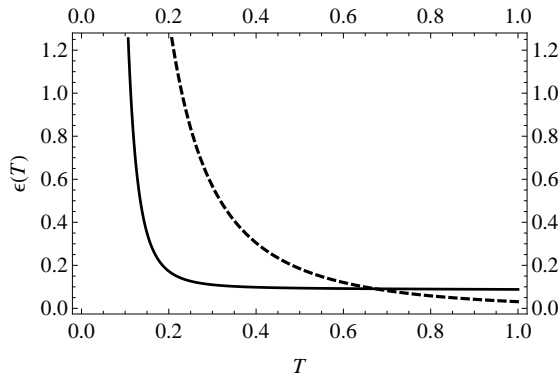


FIG. 1: Energy density of scalar field  $\epsilon_\varphi(t)$  and matter  $\epsilon_M(t)$  dependence on time. Continuous line corresponds to energy density of scalar field  $\epsilon_\varphi(t)$  and dashed line corresponds to energy density of matter  $\epsilon_M(t)$ . The point  $T = 0$  corresponds to the initial singularity and the point  $T = 1$  corresponds to the present time for all dependence on time plots.

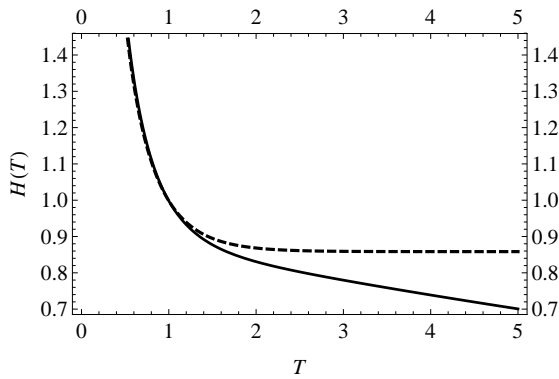


FIG. 2: Hubble parameter  $H(t)$  dependence on time. Continuous line corresponds to Hubble parameter which follows from our model and dashed line corresponds to  $\Lambda$ CDM Hubble parameter.

past than  $\Lambda$ CDM model. On the other hand  $H(t)$  dependence shows us that Hubble parameter of  $\Lambda$ CDM model will be constant when the contribution of energy density of matter can be neglected. This state of Universe will be corresponds to de-Sitter Universe in which the energy density of  $\Lambda$ -term only presents. Our model, on the contrary, gives a different result – Hubble parameter decreases with time.

The figure 3 shows that the present value of deceleration parameter  $q(t)$  which follows from our model is in good agreement with the value, which follows from  $\Lambda$ CDM model. Our model gives  $q = -0.574$  and  $\Lambda$ CDM model gives  $q = -0.61$  for  $\Omega_M = 0.26$  and  $\Omega_\Lambda = 0.74$ . The moment when the Universe stopped slowing down and began to accelerate  $q = 0$  corresponds to  $z = 0.767$  for our model and  $z = 0.785$  for  $\Lambda$ CDM model. When  $q$  is selected the parameter  $\omega_\varphi$  can be obtained from expression (6). It is important to note that parameter  $\omega_\varphi$  grows and begins positive, that leads to deceleration of

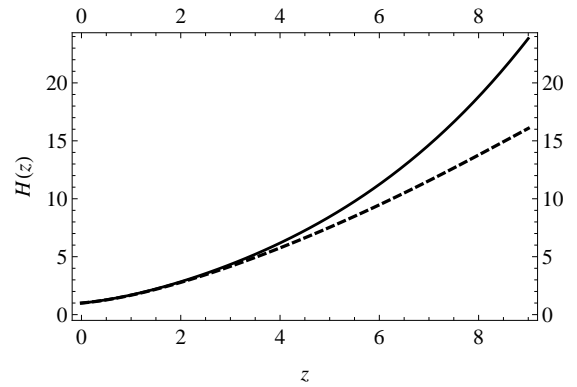


FIG. 3: Hubble parameter  $H(z)$  dependence on red-shift. Continuous line corresponds to Hubble parameter which follows from our model and dashed line corresponds to  $\Lambda$ CDM Hubble parameter.

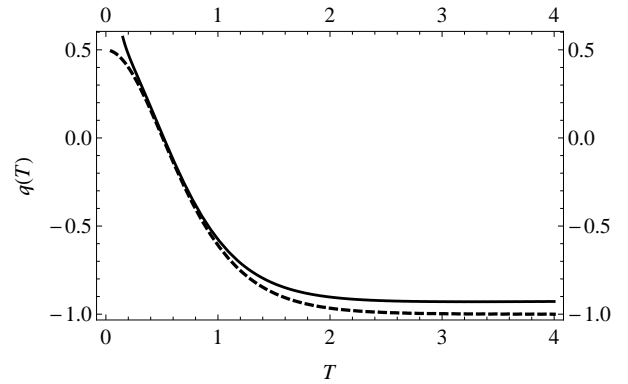


FIG. 4: The deceleration parameter  $q(t)$  dependence on time. Continuous line corresponds to deceleration parameter which follows from our model and dashed line corresponds to  $\Lambda$ CDM deceleration parameter.

cosmological expansion.

The figure 6 shows that the plots of both model overlapping in the red-shift scale from  $z = 0$  to  $z = 1.5$ . It means that our model in good agreement with the supernova observational data like  $\Lambda$ CDM model. On the other hand figure 7 of large red-shift scale shows the differences between the models. In our model the supernova are brighter than in the  $\Lambda$ CDM model at the same large red-shift. This means that in our model supernova are closer than in the  $\Lambda$ CDM model.

## V. CONCLUSION

In present work the nonlinear scalar field interacting with the gravitational field and matter is introduced. The requirement that the source of the field is the trace of its own stress-energy tensor leads to the Lagrangian containing three arbitrary parameters. An example of numerical solution of the full system of field equations, describing the cosmological solution for the homogenous

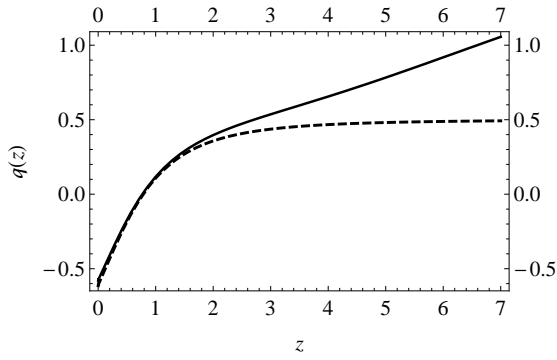


FIG. 5: The deceleration parameter  $q(z)$  dependence on redshift. Continuous line corresponds to deceleration parameter which follows from our model and dashed line corresponds to  $\Lambda$ CDM deceleration parameter.

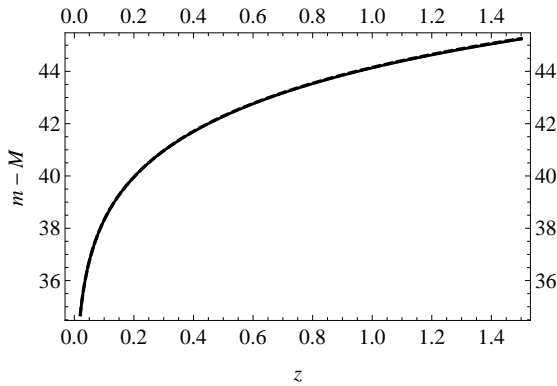


FIG. 6: The luminosity distance  $D_{lum}(z)$  dependence on redshift. Continuous line corresponds to the luminosity distance which follows from our model and dashed line corresponds to  $\Lambda$ CDM the luminosity distance. This plot shows the luminosity distance curves from  $z = 0$  to  $z = 1.5$  red-shift scale.

and isotropic Universe, shows that for certain restrictions on the parameters the slow-roll regime is possible. In this regime the scalar field simulates the dark energy in agreement with the modern observational data. More detailed analysis allowing for scalar field mass and interaction with the matter for different stages of cosmological expansion will be carry out in future.

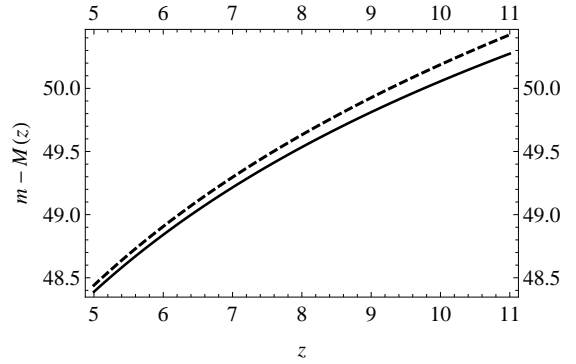


FIG. 7: The luminosity distance  $D_{lum}(z)$  dependence on redshift. Continuous line corresponds to the luminosity distance which follows from our model and dashed line corresponds to  $\Lambda$ CDM the luminosity distance. This plot shows the luminosity distance curves from  $z = 5$  to  $z = 11$  red-shift scale.

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